Estimation of the debt beta of the bond issued by Nats (En-Route) plc

By

Professor Ania Zalewska

Centre for Governance, Regulation and Industrial Strategy

School of Management, University of Bath

April 2019

Summary

This report has been written at the request of NERA Economic Consultants (NERA). Its aim is to estimate the market risk of the bond issued by Nats (En-Route) plc on 18 August 2003, ticker ED1004032 Corp (henceforth the NATS-bond). In particular, the aim is to assess whether there is any evidence that the market risk of NATS-bond is higher than 0.1. All the data used in this report were provided by NERA. The analysis covers the period August 2003 – February 2019.

The analysis was performed within the CAPM framework using OLS, ML with GARCH(1,1) effects and Kalman Filter estimates. In order to assess the market risk of the NATS-bond several alternative definitions of the market portfolio, as well as of the period of assessment, were adopted. In addition, to test reliability of the estimates, the market risk of six bonds issued by Heathrow Funding Ltd. were analysed. Heathrow Funding Ltd. was chosen because it belongs to the same sector as Nats (en-Route) plc. The six bonds issued by Heathrow Funding Ltd were selected because of the similarity of the issuance characteristics to those of the NATS-bond.

The results strongly support the thesis that the NATS-bond's beta is statistically significantly negative for most of the investigated period, and statistically insignificantly different from zero in the last few years. These results are robust across various specifications and methods of estimation. Statistically insignificant from zero betas indicate, according to the CAPM, that the returns on the NATS-bond are determined by the risk free-rate and the idiosyncratic risk of the bond (e.g. default risk) but not the performance of (returns on) the market portfolio.

Basic theoretical concepts

The assessment of debt beta may seem a straightforward exercise, but it rarely is. The methodology commonly applied to assess the market risk of equity is, theoretically speaking, suitable for the assessment of the market risk of debt. However, there are a few potential caveats that need to be taken care of to make the analysis meaningful, although the degree to which these matter may differ between markets and between periods within a market.

According to the Capital Pricing Model (CAPM), the share price risk of a publicly traded company can be decomposed into market and company specific risk. Moreover, only market risk can be expected to be 'priced' as it is undiversifiable within assets defining the market.

The CAPM states that that there is a linear relationship between returns R_i on a company i and returns on the market, R_M, i.e.

$$R_i = \alpha + \beta (R_M - R_f) + \varepsilon$$

where R_f is the risk-free rate of return, ε is the error of measurement, and α and β are parameters. One of the requirements of this relationship is that company i is included in the calculation of the returns of the market portfolio, and that its contribution to the market portfolio's returns is proportionate to the size (i.e. capitalisation) of the company in relation to the entire market.

If investors invest in equites listed on a given stock market, R_M should include of all the equities available to them that are listed on that market. If they invest in various asset classes, e.g. equities and bonds, R_M should consist of all the equities and all the bonds available to them within the specified market. Moreover, each of the components of the market portfolio should contribute to the portfolio proportionately to its size.

Thus, if γ is the ratio of the capitalisation of all the equities available to the investors to the sum of the capitalisation of these equities and of the bonds, and if one treats the equities as one asset class and the bonds as a separate asset class, then, the return on the portfolio can be expressed as $R_M = \gamma R_E + (1-\gamma)R_B$. In practice, not all investors track the equity and the bond indices, but given that these two strands of the market have readily available indices, the return on the market portfolio are often proxied by the weighted average of the returns on the equity and on the bond indices.

However, given the differences in equity and debt taxation, it seems appropriate to modify the above formula. To simplify the calculations and to make them internationally comparable, let us assume that it is only the corporate tax, T, that needs to be accounted for. Then, the CAPM equation can be expressed as:

$$R_i = \alpha + \beta (R_M - R_f(1 - (1-\gamma)T) + \varepsilon.$$

It is, however, common to simplify this specification and drop the adjustment for the bond market share, or entirely drop the tax adjustment. It is also common to use an equity index as a proxy for the market portfolio, i.e. completely ignore the bond share of the market. Although such simplifications may seem quite crude, they may not be entirely without justification. In the empirical sense, at least. This is because, the bond markets, in many countries or regions (like the European market, for example) are so small in comparison with the size of the equity markets, and the volatility of bond returns is considerably lower than the volatility of equity

returns. Thus, the adjustment for the bond market share does not make a material difference for the estimates of the beta coefficient.

Data selection

The purpose of this study is to assess the beta of the bond issued by Nats (En-Route) plc on 18 August 2003, ticker ED1004032 Corp. The basic issuance characteristics of the bond are specified in Table 1.

Table 1. Issuance characteristics of NATS-bond

Ticker	Adopted acronym	Cpn	Issue Date	Maturity Date	SP Initial Rtg	Amount Issued
ED1004032 Corp	NATS-bond	5.25	18/08/2003	31/03/2026	AAA	600000000

To test the robustness of the findings several bonds issued by Heathrow Funding Ltd. are also studied. The bonds issued by Heathrow Funding Ltd. were selected because Heathrow Funding Ltd. belongs to the same sector as NATS (En-Route) plc. Among the bonds issued by Heathrow Funding Ltd. those that have a comparable maturity to the NATS-bond and have been rated above BBB were selected. Six bonds issued by Heathrow Funding Ltd. satisfied these criteria. Their main issuance characteristics are shown in Table 2.

Table 2. Issuance characteristics of HTHRW-bonds.

Ticker	Adopted acronym	Cpn	Issue Date	Maturity Date	SP Initial Rtg	Amount Issued
EH5179385 Corp	HTHRW-bond1	5.225	18/08/2008	15/02/2023	A-	749600000
EI0650552 Corp	HTHRW-bond2	6.75	03/12/2009	03/12/2026	A-	700000000
EI0682746 Corp	HTHRW-bond3	3.334	09/12/2009	09/12/2039	A-	460000000
EH5179583 Corp	HTHRW-bond4	9.2	18/08/2008	29/03/2021	A-	249810000
EH5177660 Corp	HTHRW-bond5	7.075	18/08/2008	04/08/2028	A-	199909000
EH5177629 Corp	HTHRW-bond6	6.45	18/08/2008	10/12/2031	A-	899967000

In addition, to gain a better understanding of the general market trends, a range of iBoxx indices are also assessed. Given that the focus is on the NATS-bond, the iBoxx non-financials A rated and the iBoxx non-financials indices are of main concern.

To estimate the betas of the bonds several indices (to proxy for the market portfolio), as well as yields on government bonds and gilts (to proxy for the risk-free rate of return) were collected. The corporate tax rate was assumed to be 20%.

First, it was assumed that the equity market portfolio can be proxied by some commonly used stock market indices. The FTSE All Share index was chosen to represent the domestic equity market portfolio. However, given the international nature of investors investing on the UK market, as well as the fact that it is common for British investors to diversify abroad, and that European assets are commonly held, two European stock market indices were also selected as the potential proxies for the equity market portfolios. These were the FTSE All Europe and the

Euro Stoxx 600 indices. These indices are calculated in Euro, thus, to make them suitable for the analysis, their returns were converted into pound sterling returns.

To proxy for the bond market portfolio, the capitalisation weighted returns on the iBoxx Sterling Non-Financials and iBoxx Sterling Financials were calculated for the UK market. This index is referred to as iBoxx-UK. To account for potential investments on the European market, the capitalisation weighted returns on the iBoxx Euro Financials and the iBoxx Euro Non-Financials were also calculated. This index is referred to as iBoxx-Europe.

The risk-free rate of return was proxied by the 5-year government bond, 10-year government bond, 1-month gilts and 3-month gilts. The last two time series were available till 28 June 2018 only.

Summary statistics

Figure 1 shows that the bonds chosen for the analysis manifest considerable similarity in the size and movement of their yields with the NATS-bond. This is particularly visible after 2009, i.e. when the markets calmed down somewhat after the 2008 Financial Crisis. Figure 2 is analogous to Figure 1 but this time the series of bond prices are plotted. As fully expected, the decrease in yields is associated with the increase in prices and vice versa.

Figure 1. Yield to maturity for the selected bonds, daily observations.

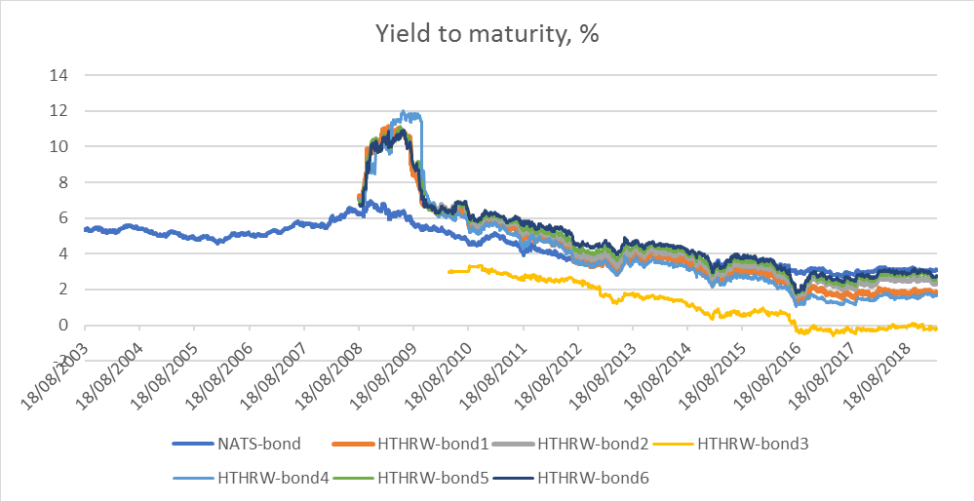


Figure 2. Price movement for the selected binds, daily observations

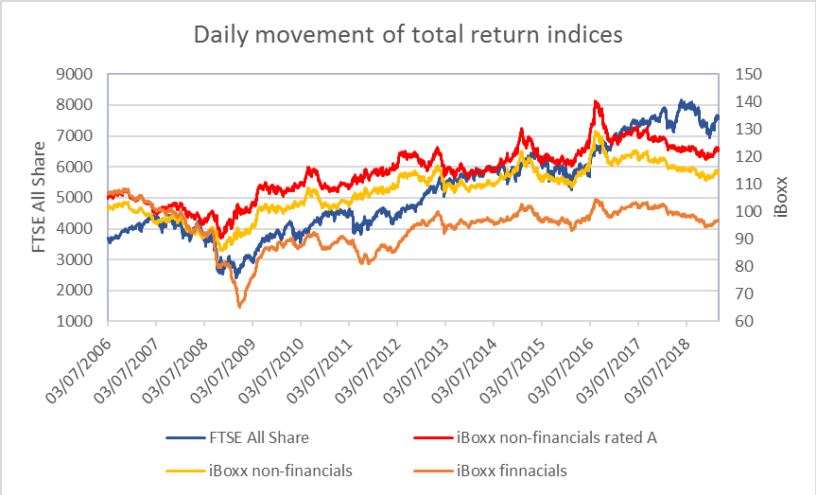
Table 3 shows the basic statistics of the time series of returns calculated for the bonds. The NATS-bond statistics are shown for the whole period of the bond's life, since 3 July 2006, i.e. from the date when the returns on the bond market indices were available, and also from 1 January 2010 to provide the assessment after the 2008 Financial Crisis and from 1 January 2016 to give an overview of the most recent developments. All the time series end on 28 February 2019. Table 3 shows that the average daily returns on the bonds were positive, that the standard deviations of the NATS-bond returns were much smaller than the standard deviations calculated for the HTHRW-bonds. Moreover, relatively high statistics obtained for HTHRW-bond4 are consistent with the bond suffering from thin trading.

Table 3. Summary statistics of the selected bonds

Bond	Obs.	Average daily return	St dev of daily returns	Minimum daily return	Maximum daily return
NATS-bond (2003-2019)	3,866	0.004	0.370	-4.551	2.519
(2006-2019)	3,256	0.002	0.370	-4.551	2.519
(2010-2019)	2,258	0.007	0.307	-2.355	1.742
(2016-2019)	782	0.000	0.182	-0.610	1.132
HTHRW-bond1	2,597	0.013	0.552	-7.690	7.615
HTHRW-bond2	2,276	0.012	0.445	-3.367	2.181
HTHRW-bond3	1,206	0.021	0.704	-2.599	6.902
HTHRW-bond4	2,595	0.003	0.893	-12.304	28.153
HTHRW-bond5	2,596	0.014	0.699	-8.918	9.748
HTHRW-bond6	2,596	0.017	0.727	-8.540	8.719

Figure 3 shows the daily movement of the FTSE All Share index (left-hand scale) and of three iBoxx Sterling indices (right-hand scale). It shows that the bond indices have very similar time-paths and these are quite different from the time-path of the FTSE All Share index. The decline of the bond indices is already observed in 2006, and the reversal starts in late 2008. The FTSE All Share and the iBoxx financials indices bounce back only in mid-2009. The FTSE All Share index doubled in its value between 2011 and 2018/2019, while the non-financials iBoxx indices increased only about 10% over the same period of time.

Figure 3. Daily movement of the FTSE All Share index and three iBoxx Sterling indices



The basic statistics of the selected and constructed indices and proxies for the risk-free return are shown in Table 4 for June 2006 – February 2019, in Table 5 for January 2010 – February 2019, and in Table 6 for 1 January 2016 – 28 February 2019.

As expected, the equity indices performed better than the bond indices, although the equity indices were also more volatile. Investments on the European equity market delivered higher returns than those earned on the domestic market when the period of the 2008 Financial Crisis was included in the calculations and in the most recent period . The recent statistics are heavily affected by the devaluation of pound sterling.

Table 4. Summary statistics of the market indices and proxies for the risk-free rate for the period 3 July 2006 – 28 February 2019.

	Obs	Average daily return	Std. Dev. of daily returns	Minimum daily return	Maximum daily return
Equity indices					
FTSE All Share	3,128	0.029	1.140	-8.341	9.211
FTSE All Europe	3,128	0.032	1.285	-8.694	9.612
Stoxx600	3,128	0.033	1.248	-8.599	9.735
Bond indices					
iBoxx-UK A rated	3,128	0.005	0.396	-1.775	2.698
iBoxx-UK non-financials	3,128	0.004	0.359	-1.564	2.330
iBoxx UK financials	3,128	-0.003	0.287	-2.508	1.541
iBoxx-UK	3,128	0.004	0.332	-1.421	2.156
iBoxx-Europe	3,128	0.000	0.170	-1.041	0.693
Equity and bond indices					
FTSE All Share + iBoxx-UK	3,128	0.027	0.965	-7.122	7.622
FTSE All Europe + iBoxx-Europe	2,916	0.017	1.104	-7.167	8.561
Stoxx600 + iBoxx-Europe	3,128	0.029	1.056	-7.450	8.162
Risk-free rate proxies					
5-year Gov Bond	3,128	0.006	0.004	0.000	0.016
10-yearGov Bond	3,128	0.008	0.004	0.002	0.015
1 month gilt	2,965	0.004	0.005	0.001	0.016
3 month gilt	2,965	0.004	0.005	0.001	0.016

Table 5. Summary statistics of the market indices and proxies for the risk-free rate for the period 1 January 2010 – 28 February 2019.

	Obs	Average daily return	Std. Dev. of daily returns	Minimum daily return	Maximum daily return
Equity indices					
FTSE All Share	2,258	0.034	0.920	-4.528	5.154
FTSE All Europe	2,258	0.033	1.095	-5.349	7.674
Stoxx600	2,258	0.035	1.064	-5.231	7.139
Bond indices					
iBoxx-UK A rated	2,258	0.006	0.386	-1.518	2.698
iBoxx-UK non-financials	2,258	0.006	0.349	-1.455	2.330
iBoxx UK financials	2,258	0.005	0.264	-1.565	1.541
iBoxx-UK	2,258	0.006	0.320	-1.409	2.156
iBoxx-Europe	2,258	0.002	0.150	-0.854	0.693
Equity and bond indices					
FTSE All Share + iBoxx-UK	2,258	0.031	0.783	-3.915	4.436
FTSE All Europe + iBoxx-Europe	2,258	0.030	0.911	-6.086	6.469
Stoxx600 + iBoxx-Europe	2,258	0.030	0.888	-4.157	5.927
Risk-free rate proxies					
5-year Gov Bond	2,260	0.004	0.002	0.000	0.008
10-yearGov Bond	2,260	0.006	0.002	0.002	0.012
1 month gilt	2,260	0.001	0.000	0.001	0.002
3 month gilt	2,260	0.001	0.000	0.001	0.002

Table 6. Summary statistics of the market indices and proxies for the risk-free rate for the period 1 January 2016 – 28 February 2019.

	Obs	Average daily return	Std. Dev. of daily returns	Minimum daily return	Maximum daily return
Equity indices					
FTSE All Share	782	0.036	1.064	-8.341	9.211
FTSE All Europe	782	0.037	1.205	-8.694	9.612
Stoxx600	782	0.039	0.863	-4.415	3.547
Bond indices					
iBoxx-UK A rated	782	0.006	0.373	-1.241	2.698
iBoxx-UK non-financials	782	0.005	0.333	-1.092	2.330
iBoxx UK financials	782	0.001	0.243	-0.777	1.541
iBoxx-UK	782	0.004	0.310	-0.994	2.156
iBoxx-Europe	782	0.000	0.122	-0.409	0.693
Equity and bond indices					
FTSE All Share + iBoxx-UK	782	0.031	0.694	-3.597	3.046
FTSE All Europe + iBoxx-Europe	782	0.019	0.805	-6.086	3.367
Stoxx600 + iBoxx-Europe	782	0.033	0.727	-3.655	2.982
Risk-free rate proxies					
5-year Gov Bond	782	0.751	0.266	0.145	1.264
10-yearGov Bond	782	1.330	0.226	0.612	1.946
1 month gilt	619	0.001	0.000	0.001	0.001
3 month gilt	619	0.001	0.000	0.001	0.001

Table 7 shows correlations for the indices used to proxy the market portfolios for 2006-2019. The equity indices are strongly correlated with each other. The bond indices show much weaker correlations with the equity indices. To save space the correlation for the other two sub-periods are not shown. This is because they are comparable to those calculated for the whole period.

Table 5 also shows that the indices constructed from the equity and the bond indices are very highly correlated with the equity indices. This suggests that regressing the bond returns on the FTSE All Share index returns and on its combination with the iBoxx-UK index will not deliver dramatically different results. These simple statistics show that the common practice of regressing returns on bonds against equity market returns, in order to estimate the debt betas, is not completely unjustified.

The negative correlations of the bond indices and of the equity or the equity-bond indices also indicate that if the bond indices themselves were regressed against the equity or the equity-bond indices, they would have negative betas.

Table 7. Correlations between the returns on the market indices.

	FTSE All Share	FTSE All Europe	Stoxx600	iBoxx-UK	iBoxx_Europe	FTSE All Share + iBoxx-UK	FTSE All Europe+ iBoxx-Europe	Stoxx600+ iBoxx-Europe
FTSE All Share	1							
FTSE All Europe	0.919	1						
Stoxx 600	0.933	0.978	1					
iBoxx-UK	-0.260	-0.241	-0.250	1				
iBoxx-Europe	-0.173	-0.190	-0.195	0.663	1			
FTSE All Share + iBoxx-UK	0.999	0.918	0.931	-0.216	-0.145	1		
FTSE All Europe + iBoxx-Europe	0.926	0.904	0.879	-0.279	-0.141	0.923	1	
Stoxx600 + iBoxx-Europe	0.934	0.977	0.999	-0.234	-0.175	0.934	0.881	1

Beta estimates

The following tables report selected results for basic OLS regressions (since these are still often used for regulatory purposes), maximum likelihood (ML) estimates with the GARCH(1,1) specification of the error terms (since these provide more accurate estimates than the OLS regressions), and finally, Kalman Filter regressions (which allow to observe time-varying pattern of the estimates). To save space only the beta coefficients and their p-values are reported for the OLS and maximum likelihood estimates. In the case of the Kalman Filter regressions the time-paths of the beta coefficient' estimates and their 95% confidence intervals are reported. The R^2-adjusted are also reported for the OLS regressions.

The results obtained for the various specifications of the risk-free return were practically indistinguishable form each other, thus, to save space, only those for the 5-year government bonds are presented.

The analysis starts from assessing the market trends using the iBoxx non-financials A rated, iBoxx non-financials and iBoxx financials indices. The analysis of the individual bonds follows.

Betas of iBoxx indices

Figure 3 shows that the FTSE All Share index and the iBoxx indices have very different patterns. Table 7 shows negative correlations for the (value weighted) bond indices and the equity market indices. Table 8 confirms that these negative correlations are a general feature of the bond indices. It shows that all three bond indices selected for the UK market were negatively correlated with the FTSE All Share index in 2006-2019. The same remains true for 2010-2016 (not reported to save space), and 2016-2019. In the 2016-2019 period the correlations weakened in comparison with those for the longer periods.

Table 8. Correlation coefficients of the daily returns for the selected UK bond indices and the FTSE All Share index

	FTSE All Share	iBoxx non-financials A rated	iBoxx non-financials	iBoxx financials
2006-2019				
FTSE All Share	1			
iBoxx non-financials A rated	-0.277	1		
iBoxx non-financials	-0.270	0.993	1	
iBoxx financials	-0.128	0.816	0.831	1
2016-2019				
FTSE All Share	1			
iBoxx non-financials A rated	-0.103	1		
iBoxx non-financials	-0.085	0.996	1	
iBoxx financials	-0.002	0.920	0.932	1

Table 9 shows OLS estimates of the betas of the UK bond indices for 2006-2019, 2010-2019 and 2016-2019, while Table 10 shows the analogous estimates for the ML estimator with GARCH(1,1) specification of the error terms.

The OLS and ML estimates are very similar confirming that (subject to the econometric correctness) both methods deliver comparable results. The differences in the estimates obtained for the non-financial A rated bonds and for all non-financial bonds are negligible. All the estimates for the non-financial indices are statistically significantly negative regardless of the period of estimation although the estimates for 2016-2019 are much smaller, in the absolute terms, than those obtained for the longer periods. The betas estimated for the iBoxx financials index are smaller (in absolute terms) than the betas of the non-financials bonds.

Table 9. OLS estimates of the beta of the bond indices against FTSE All Share index. P-values are presented in parenthesis. *** - 1%, **- 5%, and * - 10% significance.

	iBoxx non-financials A rated	iBoxx non-financials	iBoxx financials
2006-2019			
FTSE All Share	-0.096***	-0.085***	-0.032***
	(0.000)	(0.000)	(0.000)
R^2_adj	0.076	0.073	0.016
2010-2019			
FTSE All Share	-0.110***	-0.096***	-0.028***
	(0.000)	(0.000)	(0.001)
R^2_adj	0.068	0.063	0.009
2016-2019			
FTSE All Share	-0.048**	-0.035*	-0.001
	(0.029)	(0.070)	(0.964)
R^2_adj	0.009	0.006	-0.001

Table 10. ML estimates with GARCH(1,1) specification of the error terms beta od the bond indices against FTSE All Share index. P-values are presented in parenthesis. *** - 1%, **- 5%, and * - 10% significance.

	iBoxx non-financials A rated	iBoxx non-financials	iBoxx financials
2006-2019			
FTSE All Share	-0.099***	-0.088***	-0.034***
	(0.000)	(0.000)	(0.000)
2010-2019			
FTSE All Share	-0.110***	-0.096***	-0.030***
	(0.000)	(0.000)	(0.000)
2016-2019			
FTSE All Share	-0.050***	-0.040***	-0.006
	(0.000)	(0.002)	(0.532)

Given that the estimates are sensitive to the period of analysis, Figure 4 shows the time-paths of the beta for the iBoxx indices estimated by the Kalman Filter. There is a great similarity of the time-paths across the indices. As the OLS and ML estimates are negative, the beta estimates obtained on a more granular basis also tend to be negative. However, in the most recent years, there are short periods of time over which the beta estimates are positive. These positive estimates are not statistically significant as Figure 5 shows. To save space Figure 5 shows the beta time-path and the 95% confidence intervals for the iBoxx non-financials index' betas only.

Figure 4. The Kalman Filter estimates of the daily betas of the iBoxx non-financials A rated index, the iBoxx non-financials index and the iBoxx financials index with the FTSE All Share index used as the proxy for the market portfolio.

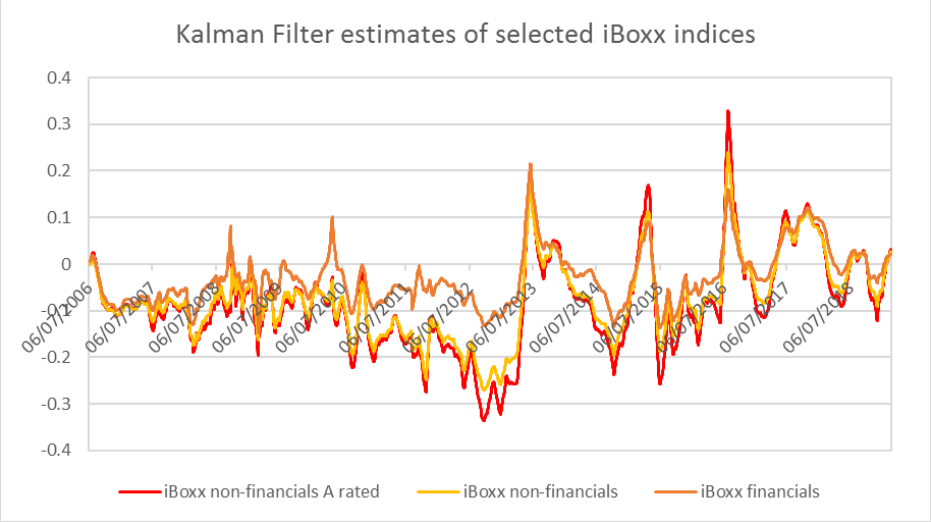
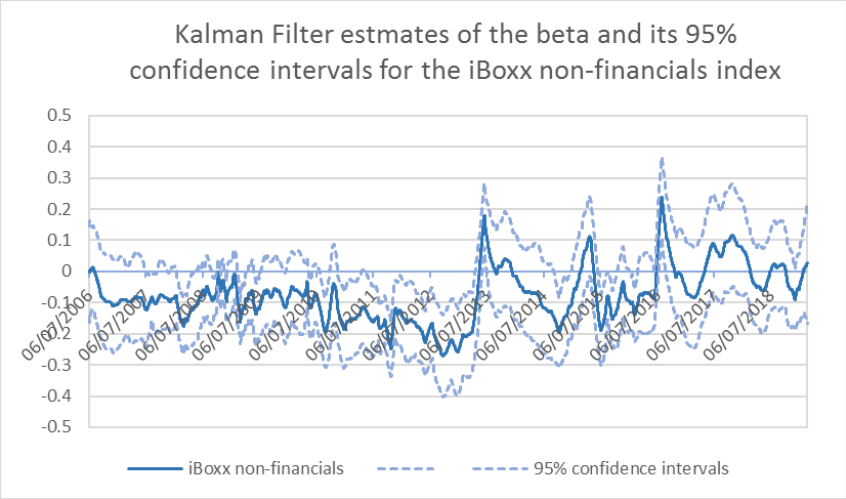


Figure 5. The Kalman Filter estimates of the daily betas of the iBoxx non-financials index with the FTSE All Share index used as the proxy for the market portfolio.



Betas of the individual bonds

Table 11 shows the OLS estimates of the betas for all the NATS-bond and the HTHRW-bonds when the market portfolio is proxied by the equity market indices. The results show great consistency across the market portfolio proxies. They are also comparable across the bonds. All the betas are statistically significantly negative.

The beta estimates of the NATS-bond are in the middle range of those obtained for the HTHRW-bond regardless of the specification of the market portfolio.

Table 11. Estimates of the betas obtained for the OLS regressions for the NATS-bond and the HTHRW 1-6 bonds against the equity indices (as specified in the first column). P-values are presented in parenthesis. *** - 1%, **- 5%, and * - 10% significance.

		HTHRW-					
	NATS-bond	bond1	bond2	bond3	bond4	bond5	bond6
FTSE All Share	-0.112***	-0.104***	-0.124***	-0.104***	-0.127***	-0.072***	-0.130***
	(0.000)	(0.000)	(0.000)	(0.002)	(0.000)	(0.009)	(0.000)
R^2adj	0.103	0.046	0.066	0.014	0.026	0.013	0.041
FTSE All Europe	-0.092***	-0.086***	-0.097***	-0.069***	-0.106***	-0.060**	-0.105***
	(0.000)	(0.000)	(0.000)	(0.008)	(0.000)	(0.010)	(0.000)
R^2adj	0.090	0.042	0.057	0.007	0.024	0.013	0.036
Stoxx 600	-0.098***	-0.093***	-0.106***	-0.075***	-0.113***	-0.072***	-0.119***
	(0.000)	(0.000)	(0.000)	(0.004)	(0.000)	(0.003)	(0.000)
R^2adj	0.095	0.046	0.064	0.009	0.26	0.017	0.044

Table 12 shows the results analogous to those presented in Table 11, but this time the returns on the bonds are regressed on the returns on the capitalisation weighted indices constructed from the equity and the bond indices minus the risk-free rate of return proxied by the 5-year government bond. Using this 'more sophisticated' proxy for the market portfolio has hardly any impact on the results.

Table 12. Estimates of the betas obtained for the OLS regressions for the NATS-bond and the HTHRW 1-6 bonds against the selected capitalisation weighted, indices constructed from the equity indices and bond indices (as specified in the first column). P-values are presented in parenthesis. *** - 1%, **- 5%, and * - 10% significance.

	NATS-bond	HTHRW-					
		bond1	bond2	bond3	bond4	bond5	bond6
FTSE All Share + iBoxx-UK	-0.121***	-0.105***	-0.119***	-0.086**	-0.139***	-0.064**	-0.129***
	(0.000)	(0.000)	(0.000)	(0.026)	(0.000)	(0.044)	(0.000)
R^2adj	0.099	0.033	0.043	0.007	0.022	0.007	0.029
FTSE All Europe + iBoxx-Europe	-0.128***	-0.109***	-0.133***	-0.145***	-0.135***	-0.081***	-0.140***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.004)	(0.000)
R^2adj	0.138	0.047	0.074	0.031	0.027	0.016	0.044
Stoxx 600 + iBoxx-Europe	-0.115***	-0.104***	-0.118***	-0.080***	-0.131***	-0.079***	-0.136***
	(0.000)	(0.000)	(0.000)	(0.009)	(0.000)	(0.007)	(0.000)
R^2adj	0.106	0.041	0.055	0.007	0.24	0.014	0.039

Table 13 shows the OLS estimates of the betas when the market portfolio is proxied by the tax-adjusted capitalisation weighted index constructed of the stock market indices and the bond indices as specified in the first column. It shows that, once more, regardless of the market portfolio specification the betas are negative and statistically significant. Moreover, the tax-adjustment does not impact on the estimates in any material way.

Table 13. Estimates of the betas obtained for the OLS regressions for the NATS-bond and the HTHRW 1-6 bonds against the selected capitalisation weighted, tax adjusted indices constructed from equity and bond indices (as specified in the first column). P-values are presented in parenthesis. *** - 1%, **- 5%, and * - 10% significance.

	NATS-bond	HTHRW-					
		bond1	bond2	bond3	bond4	bond5	bond6
FTSE All Share + iBoxx-UK	-0.121***	-0.105***	-0.119***	-0.086**	-0.139***	-0.064**	-0.129***
	(0.000)	(0.000)	(0.000)	(0.026)	(0.000)	(0.044)	(0.000)
R^2adj	0.099	0.033	0.043	0.007	0.022	0.007	0.029
FTSE All Europe + iBoxx-Europe	-0.128***	-0.109***	-0.133***	-0.145***	-0.135***	-0.081***	-0.140***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.004)	(0.000)
R^2adj	0.138	0.047	0.075	0.031	0.027	0.016	0.044
Stoxx 600 + iBoxx-Europe	-0.115***	-0.104***	-0.118***	-0.080***	-0.131***	-0.079***	-0.136***
FTSE All Share + iBoxx-UK	(0.000)	(0.000)	(0.000)	(0.009)	(0.000)	(0.007)	(0.000)
	0.106	0.41	0.055	0.007	0.024	0.014	0.039

These results are robust to various changes in the specification and the estimation method (i.e. OLS v ML). To save space Table 14 shows the results analogous to those presented above, but when the OLS is replaced by the ML estimator with the GARCH(1,1) specification of the error terms and the period of estimation is reduced to 1 January 2010 – 28 February 2019 to exclude any potential effect of the financial crisis. There are hardly any changes to the size of the estimates and none to their significance.

Table 14. Estimates of the betas obtained for the ML regressions with GARCH(1,1) specification of the error terms for the NATS-bond and the HTHRW 1-6 bonds against the selected equity market indices (as specified in the first column) (Panel A), capitalisation weighted indices constructed from the equity indices and bond indices (without the tax-adjustment) (Panel B) and the tax-adjusted capitalisation weighted indices constructed from the equity indices and bond indices (Panel C). P-values are presented in parenthesis. *** - 1%, **- 5%, and * - 10% significance. The period of the regressions:01 January 2010– 28 February 2019.

		HTHRW-					
	NATS-bond	bond1	bond2	bond3	bond4	bond5	bond6
Panel A							
FTSE All Share	-0.108***	-0.087***	-0.117***	-0.097***	-0.064***	-0.123***	-0.126***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
FTSE All Europe	-0.088***	-0.070***	-0.092***	-0.072***	-0.057***	-0.096***	-0.097***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Stoxx 600	-0.095***	-0.076***	-0.102***	-0.078***	-0.062***	-0.106***	-0.109***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Panel B							
FTSE All Share + iBoxx-UK	-0.109***	-0.081***	-0.108***	-0.081***	-0.059***	-0.115***	-0.116***
	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)
FTSE All Europe + iBoxx-Europe	-0.120***	-0.099***	-0.129***	-0.144***	-0.074***	-0.135***	-0.138***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Stoxx 600 + iBoxx-Europe	-0.108***	-0.084***	-0.112***	-0.084***	-0.068***	-0.118***	-0.120***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Panel C							
FTSE All Share + iBoxx-UK	-0.109***	-0.081***	-0.108***	-0.081***	-0.059***	-0.115***	-0.116***
	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)
FTSE All Europe + iBoxx-Europe	-0.120***	-0.099***	-0.129***	-0.144***	-0.074***	-0.135***	-0.138***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Stoxx 600 + iBoxx-Europe	-0.108***	-0.084***	-0.112***	-0.084***	-0.068***	-0.118***	-0.120***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

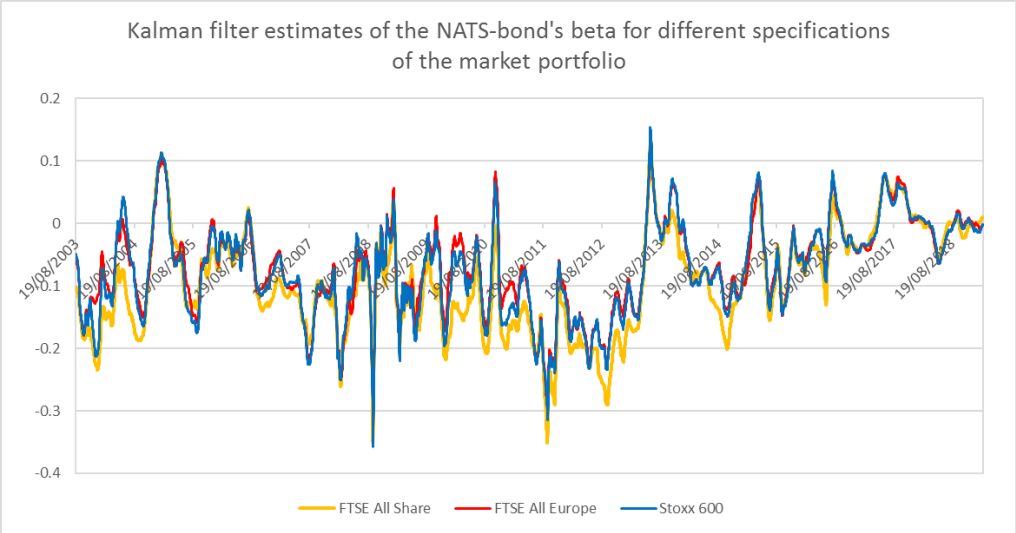
Table 15 confirms that weakening of the correlations between the bond and the equity indices observed in the last few years has had an impact on the betas. All the betas estimated for 2016-2019 are smaller in their absolute terms than those obtained for 2010-2019. Moreover, some of the HTHRW estimates are no longer statistically significant.

Table 15. Estimates of the betas obtained for the ML regressions with GARCH(1,1) specification of the error terms for the NATS-bond and the HTHRW 1-6 bonds against the selected equity market indices (as specified in the first column) (Panel A), capitalisation weighted indices constructed from the equity indices and bond indices (without the tax-adjustment) (Panel B) and the tax-adjusted capitalisation weighted indices constructed from the equity indices and bond indices (Panel C). P-values are presented in parenthesis. *** - 1%, **- 5%, and * - 10% significance. The period of the regressions:01 January 2016– 28 February 2019.

		HTHRW-					
	NATS-bond	bond1	bond2	bond3	bond4	bond5	bond6
Panel A							
FTSE All Share	-0.026***	-0.026***	-0.044***	-0.038	-0.011***	-0.048***	-0.052***
	(0.000)	(0.000)	(0.000)	(0.155)	(0.002)	(0.000)	(0.001)
FTSE All Europe	-0.014**	-0.012**	-0.024**	-0.008	-0.002	-0.029***	-0.031**
	(0.012)	(0.025)	(0.016)	(0.729)	(0.470)	(0.009)	(0.024)
Stoxx 600	-0.017***	-0.014***	-0.028***	-0.020	-0.003	-0.033***	-0.038***
	(0.003)	(0.009)	(0.007)	(0.420)	(0.331)	(0.005)	(0.010)
Panel B							
FTSE All Share + iBoxx-UK	-0.030***	-0.018***	-0.031**	-0.013	-0.006	-0.034**	-0.033*
	(0.000)	(0.008)	(0.015)	(0.664)	(0.155)	(0.014)	(0.052)
FTSE All Europe + iBoxx-Europe	-0.053***	-0.050***	-0.080***	-0.105***	-0.027***	-0.090***	-0.102***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Stoxx 600 + iBoxx-Europe	-0.017**	-0.014**	-0.027**	-0.016	-0.002	-0.032**	-0.036**
	(0.013)	(0.034)	(0.025)	(0.600)	(0.614)	(0.016)	(0.034)
Panel C							
FTSE All Share + iBoxx-UK	-0.030***	-0.018***	-0.031**	-0.013	-0.006	-0.034**	-0.033*
	(0.000)	(0.008)	(0.015)	(0.664)	(0.155)	(0.014)	(0.052)
FTSE All Europe + iBoxx-Europe	-0.053***	-0.050***	-0.080***	-0.105***	-0.027***	-0.090***	-0.102***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Stoxx 600 + iBoxx-Europe	-0.017**	-0.014**	-0.027**	-0.016	-0.002	-0.032**	-0.036**
	(0.013)	(0.034)	(0.025)	(0.600)	(0.614)	(0.016)	(0.034)

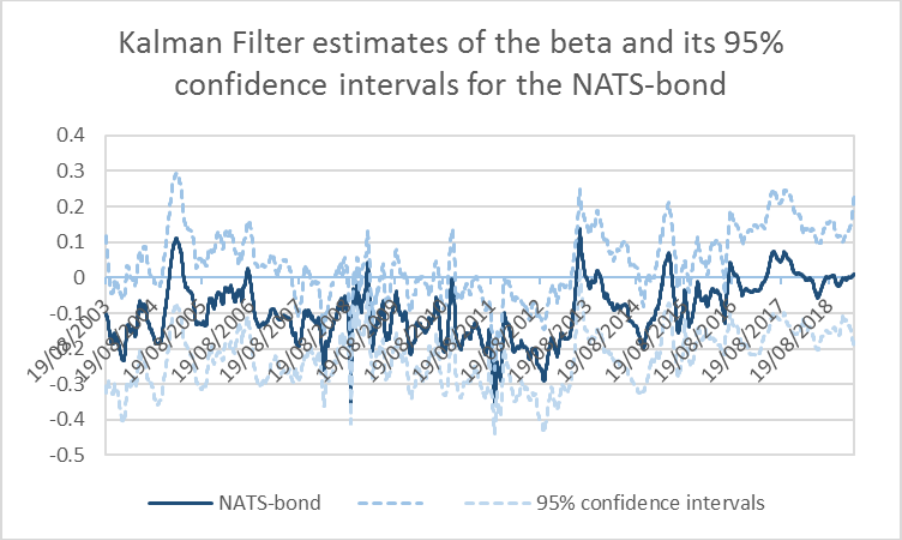
The changes in the size and significance of the estimates call for a closer look at the time variation of the betas. Figure 6 shows the Kalman Filter estimates of the daily movement of the betas for the NATS-bond when the market portfolio is proxied by the FTSE All Share index (yellow line), by the FTSE All Share Europe (red line) and the Stoxx 600 index (blue line). It confirms the previous results, i.e. (i) there is a great similarity between the estimates obtained for the three specifications of the market portfolio, (ii) the most recent betas are much closer to zero than the estimates obtained for the 2008-2014 period, and (iii) with an exception of a few short-lived 'spikes' the betas are non-positive and range between -0.2 and 0.

Figure 6. Daily estimates of the beta for NATS-bond when the market portfolio is as the legend specifies.



As Figure 6 is already 'dense' even without the time-paths of the confidence intervals of the estimated betas, Figure 7 provides the missing information, i.e. it shows the beta time-path and its 95% confidence intervals for the betas obtained from the regressions using the FTSE All Share index as the proxy for the market portfolio. The confidence intervals estimated for the other beta paths are very similar, so to save space, these are not shown. Figure 7 shows that the most recent estimates are statistically insignificant. The time-pattern of the beta estimated for the NATS-bond resembles the time-pattern estimated for the iBoxx indices as shown in Figures 4 and 5.

Figure 7. Daily estimates of the beta (solid line) and its 95% confidence intervals (dotted lines) for NATS-bond when the market portfolio is proxied by the FTSE All Share index.



To complete the discussion, Figures 8 and 9 show the time-paths of the beta estimated for the HTHRW-bonds using the FTSE All Share index as the market portfolio. Figure 8 shows that there is a great similarity across the six HTHRW-bonds' time paths. Figure 9 confirms that the time-path estimated for the NATS-bond is comparable with those of the HTHRW-bonds. It plots the average of the HTHRW-bonds' time paths and the time-path of NATS-bond. The betas show considerable time-variations and the current values are smaller than those obtained for the 2008 Financial Crisis period. Thus, the pattern observed for NATS-bond is not NATS-bond specific.

Figure 8. Daily estimates of the beta for HTHRW-bonds when the market portfolio is proxied by the FTSE All Share index.

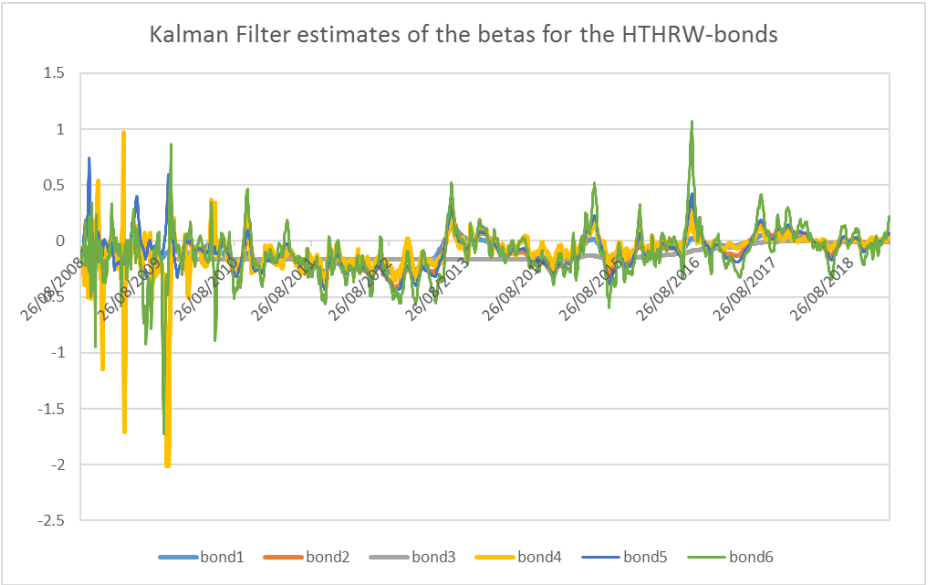
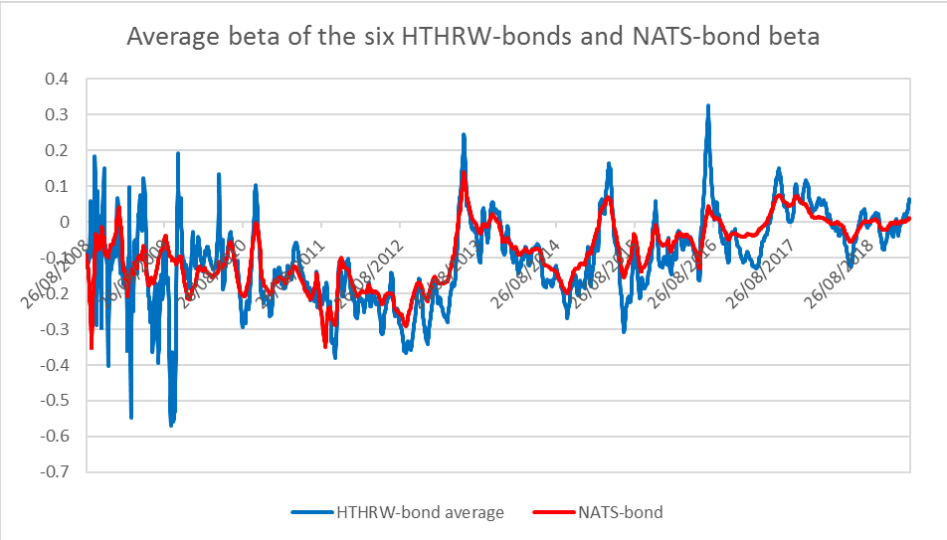


Figure 9. The average of the Kalman Filter estimates of the beta for the HTHRW-bonds and the Kalman Filter beta estimate for the NATS-bond.



Weekly/monthly data

Daily observations pick up short-term fluctuations even if they present a very accurate and precise state of affairs. To provide a smoother picture of the trends, it is common to lower the frequency of the data. However, lowering the data frequency is burdened with the loss of precision of the estimates and higher standard errors. Thus, while lowering the data frequency may help to see 'the bigger picture' one should be aware of its side effects.

Below, the results for weekly data are discussed. Weekly data (from Monday to Monday) use only one in five observations that daily data would use. They are preferred to the monthly observations that would select only one in 22-23 observations for the analysis.

Figure 10 is analogous to Figure 6, but this time weekly returns are used to estimate the time-paths of the beta. It shows, once more, that the estimates are robust to the different specifications of the market portfolio, but more importantly, it confirms the non-positiveness of the beta observed for the daily data. Figure 11 (analogous to Figure 7) confirms this observation. It plots the beta estimated against the FTSE All Share index and the 95% confidence intervals obtained for these estimates. It confirms that during the 2008 Financial Crisis and the Sovereign Debt crisis, the betas were statistically significantly negative, but in the most recent years they became statistically insignificant. Moreover, statistically speaking, the hypothesis that the beta is equal or higher than 0.1 can be rejected with 95% confidence.

Figure 10. Weekly estimates of the beta for NATS-bond when the market portfolio is as the legend specifies.

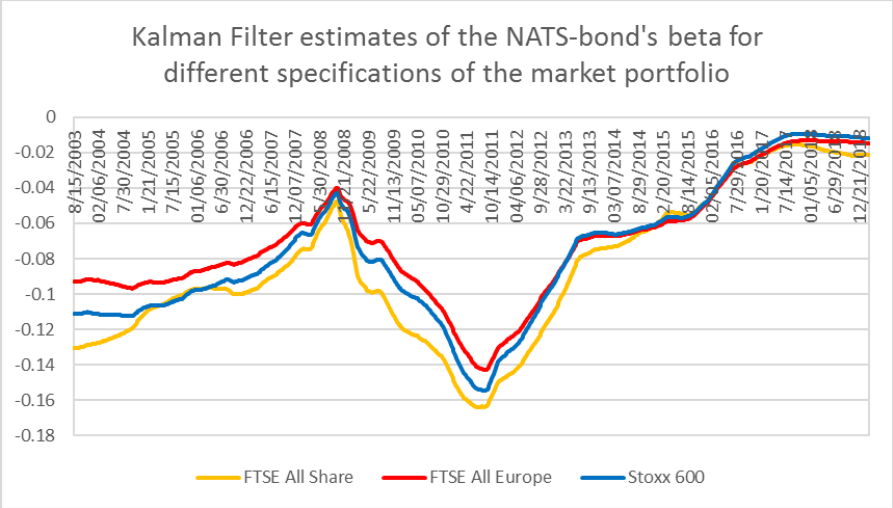


Figure 11. Weekly estimates of the beta (solid line) and its 95% confidence intervals (dotted lines) for NATS-bond when the market portfolio is proxied by the FTSE All Share index.

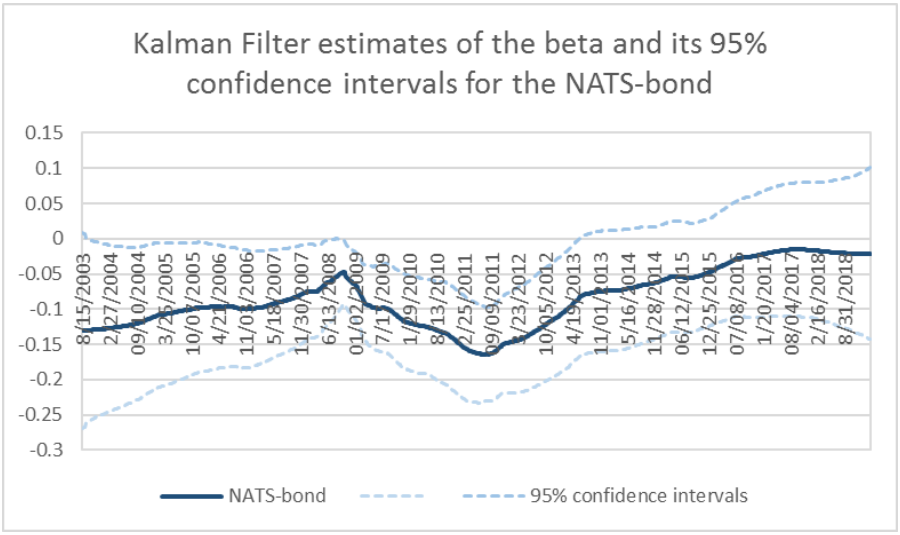


Figure 12 (analogous to Figure 9) compares the beta estimates of the NATS-bond and the average of the betas estimated for the HTHRW-bonds. Once more, it can be seen that the beta of the NATS-bond is more stable than the HTHRW-bonds' average. Moreover, the two time-paths show a considerable similarity with lower values observed in 2008-2014, and higher values, but still close to zero in the more recent years.

Figure 12. The average of the Kalman Filter estimates of the beta for the HTHRW-bonds and the Kalman Filter beta estimate for the NATS-bond.

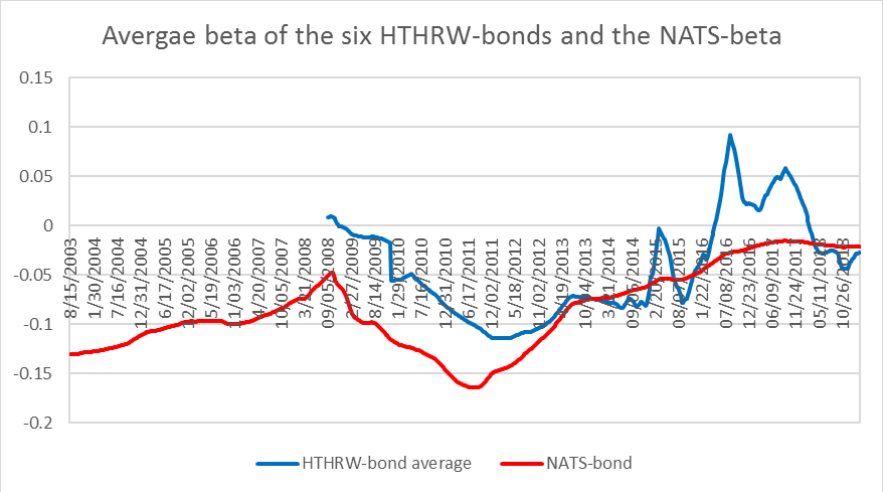


Figure 13 (analogous to Figure 4) compares the time-path of the betas for the main iBoxx indices. Figure 14 (analogous to Figure 5), once more confirms the trends observed for the individual bonds estimated for the weekly data and for the daily data. While in the most recent years the betas have increased, they remain statistically indifferent from zero and 0.1, but statistically significantly different from 0.2.

Figure 13. The Kalman Filter estimates of the weekly betas of the iBoxx non-financials A rated index, the iBoxx non-financials index and the iBoxx financials index with the FTSE All Share index used as the proxy for the market portfolio.

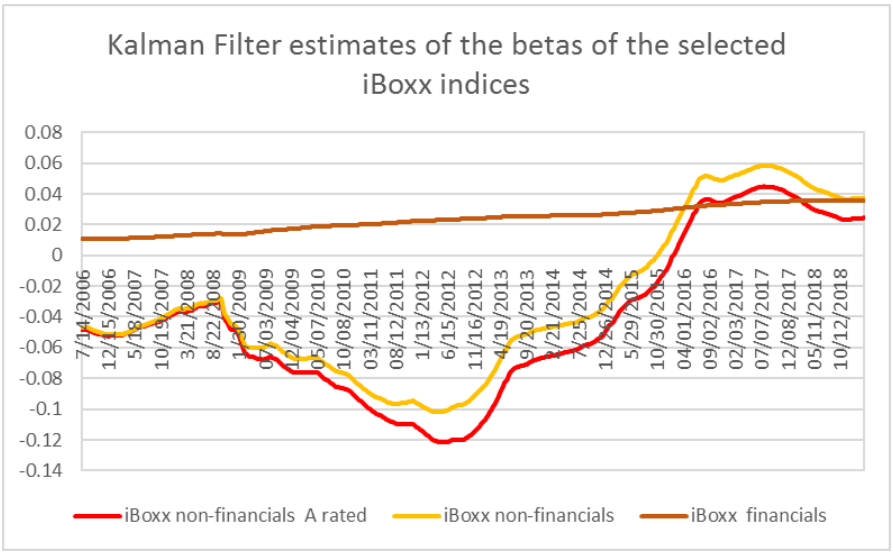


Figure 14. The Kalman Filter estimates of the weekly betas of the iBoxx non-financials index with the FTSE All Share index used as the proxy for the market portfolio.

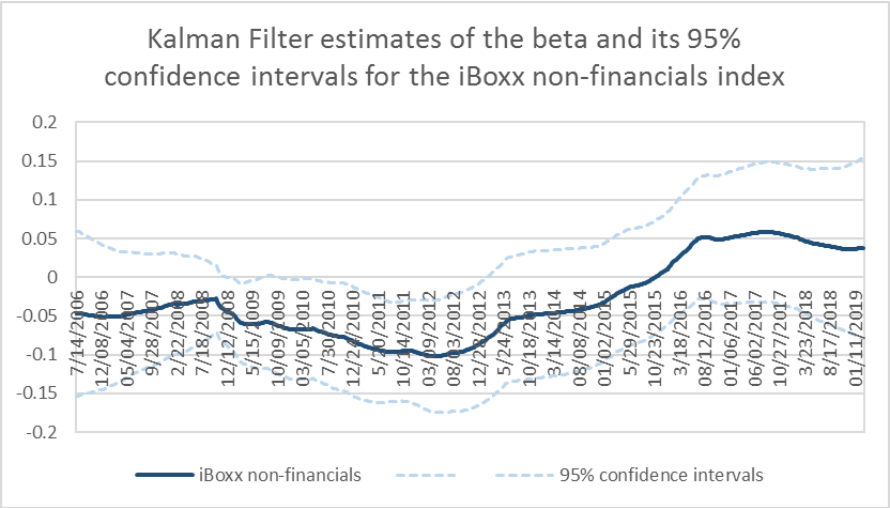


Table 16 shows the summary of the OLS regression estimates of the NATS-bond betas for the weekly data. The first column with the results shows the estimates for the 2010-2019 period. The following two columns shows the estimates of the regressions that used a time dummy for the 2016-2019 period to test the significance of the change in the beta in 2016-2019. If there is a statistically significant change in the beta in 2016-2019, the coefficient estimated for the 2016-2019 dummy will be statistically significant. The beta for 2016-2019 is the sum of the estimated beta and of the estimated dummy. The last column shows the estimates of the beta for the 2016-2019 period.

The estimates show great similarity to those presented in Tables 11-13. The betas are statistically significantly negative in 2010-2019. Controlling for the potential change in the beta, shows that, indeed, in the most recent years the betas increased. However, the coefficients estimated for the 2016-2019 dummy are always smaller than the estimates for the betas. This means that in total (after adding the beta estimate and the dummy estimate) the beta for the 2016-2019 period is not bigger than zero. This is confirmed by the estimates presented in the last column. When the estimation period is limited to the 2016-2019 period, every single beta estimate is statistically insignificantly different from zero. Moreover, they are statistically significantly smaller than 0.1. Betas that are statistically insignificant from zero indicate, according to the CAPM, that the returns on the NATS-bond are determined by the risk free-rate and the idiosyncratic risk of the bond (e.g. default risk) but not by the performance of (returns on) the market portfolio.

These results are confirmed by Table 17 where ML estimates with GARCH effects are presented. The estimated coefficients tend to be smaller than those obtained for the OLS regressions (Table 16), but the general pattern is preserved. That is, the 2010-2016 betas are negative and the 2016-2019 betas are statistically insignificantly different from zero and statistically significantly smaller than 0.1.

Table 16. Estimates of the betas obtained for the OLS regressions for the NATS-bond against stock market indices (Panel A), the capitalisation weighted indices constructed from the equity and the bond indices (Panel B), and tax-adjusted, capitalisation weighted indices constructed from the equity and the bond indices (Panel C). The equity and the bond indices are specified in the first column. P-values are presented in parenthesis. *** - 1%, **- 5%, and * - 10% significance.

	2010-2019	2010-2019		2016-2019
	beta	beta	2016-2019 dummy	beta
Panel A				
FTSE All Share	-0.096***	-0.124***	0.111***	-0.010
	(0.000)	(0.000)	(0.001)	(0.687)
R^2adj	0.068		0.084	-0.005
FTSE All Europe	-0.088***	-0.114***	0.108***	-0.003
	(0.000)	(0.000)	(0.000)	(0.888)
R^2adj	0.073		0.092	-0.006
Stoxx 600	-0.091***	-0.118***	0.112***	-0.003
	(0.000)	(0.000)	(0.000)	(0.878)
	0.078		0.098	-0.006
Panel B				
FTSE All Share + iBoxx-UK	-0.066***	-0.071***	0.044*	0.008
	(0.000)	(0.000)	(0.076)	(0.746)
R^2adj	0.036		0.038	-0.005
FTSE All Europe + iBoxx-Europe	-0.064***	-0.071***	0.056**	0.018
	(0.000)	(0.000)	(0.022)	(0.424)
R^2adj	0.040		0.046	-0.001
Stoxx 600 + iBoxx-Europe	-0.067***	-0.075***	0.056**	0.016
FTSE All Share + iBoxx-UK	(0.000)	(0.000)	(0.019)	(0.497)
	0.046		0.052	-0.002
Panel C				
FTSE All Share + iBoxx-UK	-0.067***	-0.072***	0.047*	0.008
	(0.000)	(0.000)	(0.066)	(0.750)
R^2adj	0.036	0.039		-0.005
FTSE All Europe + iBoxx-Europe	-0.071***	-0.082***	0.073***	0.018
	(0.000)	(0.000)	(0.006)	(0.439)
R^2adj	0.044		0.053	-0.001
Stoxx 600 + iBoxx-Europe	-0.073***	-0.084***	0.071***	0.015
FTSE All Share + iBoxx-UK	(0.000)	(0.000)	(0.007)	(0.519)
R^2adj	0.049		0.058	-0.003

Table 17. Estimates of the betas obtained for the ML regressions for the NATS-bond against stock market indices (Panel A), the capitalisation weighted indices constructed from the equity and the bond indices (Panel B), and tax-adjusted, capitalisation weighted indices constructed from the equity and the bond indices (Panel C). The equity and the bond indices are specified in the first column. P-values are presented in parenthesis. *** - 1%, **- 5%, and * - 10% significance.

	2010-2019	2010-2019		2016-2019
	beta	beta	dummy	beta
Panel A				
FTSE All Share	-0.062***	-0.112***	0.097***	0.003
	(0.000)	(0.000)	(0.001)	(0.863)
FTSE All Europe	-0.057***	-0.108***	0.103***	-0.003
	(0.000)	(0.000)	(0.000)	(0.838)
Stoxx 600	-0.058***	-0.111***	0.107***	-0.004
	(0.000)	(0.000)	(0.000)	(0.826)
Panel B				
FTSE All Share + iBoxx-UK	-0.038***	-0.051***	0.040**	0.008
	(0.003)	(0.000)	(0.046)	(0.641)
FTSE All Europe + iBoxx-Europe	-0.035***	-0.053***	0.053***	0.018
	(0.004)	(0.000)	(0.005)	(0.294)
Stoxx 600 + iBoxx-Europe	-0.037***	-0.056***	0.054***	0.015
FTSE All Share + iBoxx-UK	(0.002)	(0.000)	(0.004)	(0.365)
Panel C				
FTSE All Share + iBoxx-UK	-0.039***	-0.052***	0.041**	0.008
	(0.003)	(0.000)	(0.041)	(0.647)
FTSE All Europe + iBoxx-Europe	-0.038***	-0.051***	0.040**	0.018
	(0.003)	(0.000)	(0.046)	(0.307)
Stoxx 600 + iBoxx-Europe	-0.040***	-0.065***	0.065***	0.015
FTSE All Share + iBoxx-UK	(0.001)	(0.000)	(0.001)	(0.385)

Finally, to show that reducing the frequency to monthly can result in unreliable outcomes, Tables 18 and 19 present results analogous to those shown in Table 16 Panel A but for differently defined monthly returns. Table 18 is based on the mid-month returns while Table 19 is based on the end-of-month returns. While regressing the returns on NATS-bond against the returns on the FTSE All Share index delivers statistically positive beta in 2016-2019, this is not confirmed by the regression using the end-of-month returns and the other stock market indices to proxy for the market portfolio.

Table 18. Estimates of the betas obtained for the OLS regressions for the NATS-bond against stock market indices using mid-month monthly data. P-values are presented in parenthesis. *** - 1%, **- 5%, and * - 10% significance.

	2010-2019	2010-2019		2016-2019
	beta	beta	2016-2019 dummy	beta
FTSE All Share	-0.069*	-0.120***	0.159***	0.065**
	(0.067)	(0.010)	(0.004)	(0.043)
R^2adj	0.035		0.042	0.041
FTSE All Europe	-0.066*	-0.108**	0.138***	0.050
	(0.057)	(0.015)	(0.010)	(0.116)
R^2adj	0.081		0.081	0.085
Stoxx 600	-0.067*	-0.113**	0.145***	0.052
	(0.060)	(0.015)	(0.008)	(0.106)
R^2adj	0.092		0.054	0.061

Table 19. Estimates of the betas obtained for the OLS regressions for the NATS-bond against stock market indices using end-of-month monthly data. P-values are presented in parenthesis. *** - 1%, **- 5%, and * - 10% significance.

	2010-2019	2010-2019		2016-2019
	beta	beta	2016-2019 dummy	beta
FTSE All Share	-0.012	-0.029	0.072	0.060
	(0.739)	(0.518)	(0.212)	(0.172)
R^2adj	-0.008		-0.002	-0.002
FTSE All Europe	-0.026	-0.036	0.056	0.036
	(0.410)	(0.319)	(0.292)	(0.404)
R^2adj	-0.009		-0.005	-0.007
Stoxx 600	-0.025	-0.035	0.052	0.033
	(0.439)	(0.356)	(0.314)	(0.417)
R^2adj	0.022		-0.008	-0.011